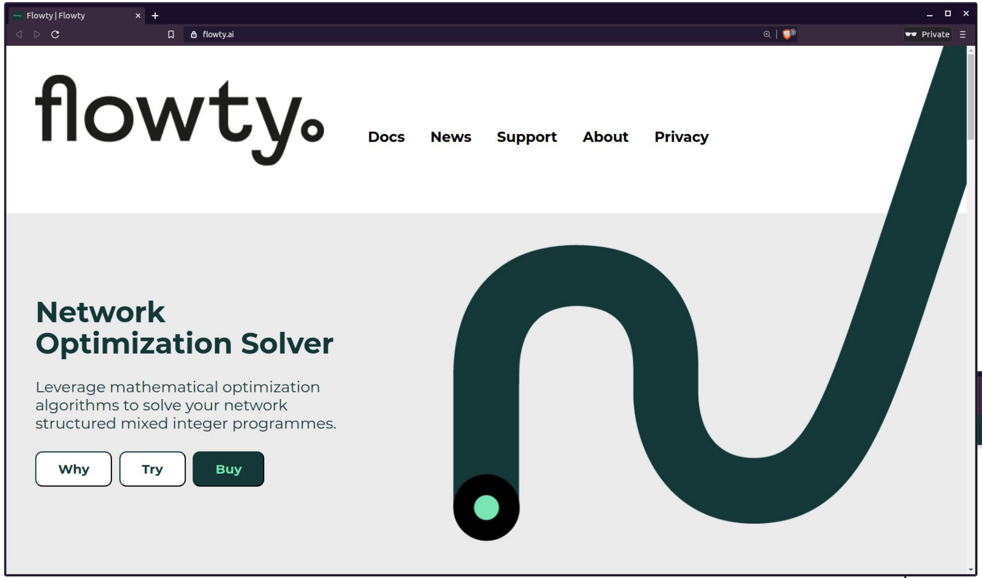
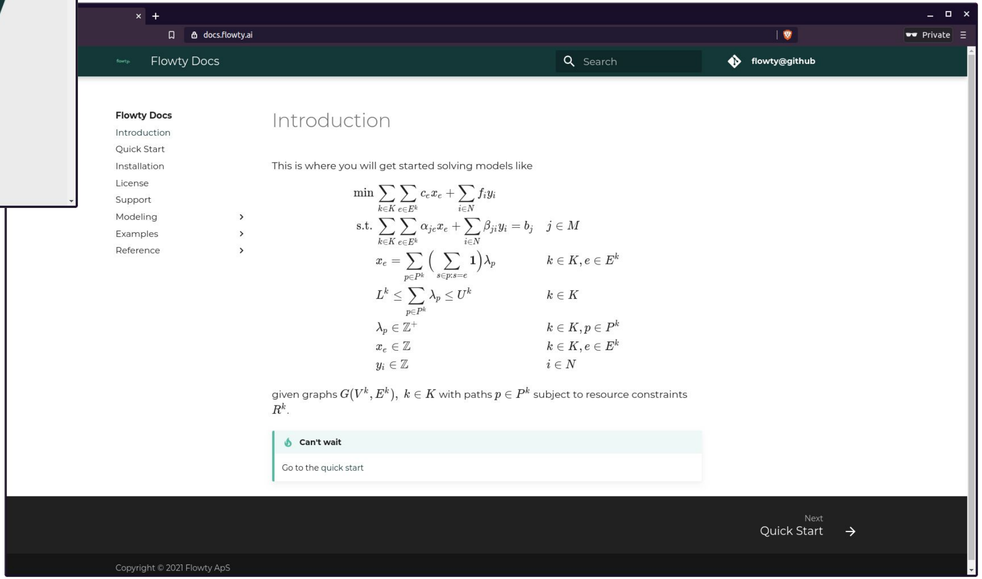
Optimization Algorithms may be the answer when Machine Learning is not

or

Why mathematical optimization and operations research make sense

flowty.





flowty.ai

docs.flowty.ai

github.com/flowty

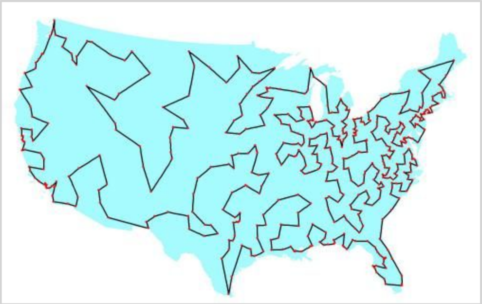
Travelling Salesman Problem

The travelling salesman problem ([...] TSP) asks the following question: "Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city and returns to the origin city?" It is an NP-hard problem ...

Source: Wikipedia

Express as a Graph

- Graph *G(V, E)*
- Vertices *V* represents cities
- Edges (i,j) *E* connects vertices
- A weight/cost c is associated with the edges



532-city USA tour; Padberg-Rinaldi 1987

source : https://www.math.uwaterloo.ca/tsp

Decode the Description

Objective:
Minimize travel distance between cities

Constraints:
i) Visit each city exactly once
ii) Return to origin city, i.e, tour must be a round-trip

Decisions:
Did the salesman travel between city i and j? Yes/no.

$$\min \sum_{i=1}^{n} \sum_{j\neq i, j=1}^{n} c_{ij} x_{ij}:$$

$$\sum_{i=1, i\neq j}^{n} x_{ij} = 1 \qquad j = 1, \ldots, n;$$

$$\sum_{j=1, j\neq i}^{n} x_{ij} = 1 \qquad i = 1, \ldots, n;$$

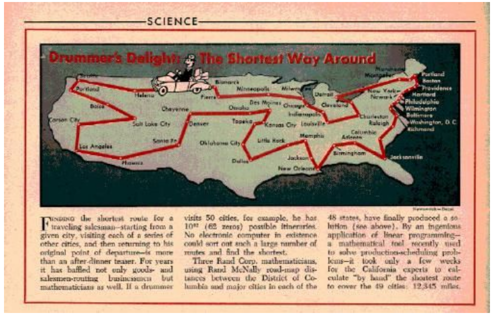
$$\sum_{i\in Q} \sum_{j\neq i, j\in Q} x_{ij} \leq |Q| - 1 \qquad \forall Q \subsetneq \{1, \ldots, n\}, |Q| \geq 2$$

$$x_{ij} = \begin{cases} 1 & \text{the path goes from city } i \text{ to city } j \\ 0 & \text{otherwise} \end{cases}$$

Dantzig–Fulkerson–Johnson formulation

Source: https://en.wikipedia.org/wiki/Travelling_salesman_problem

Current TSP status



SCIENCE

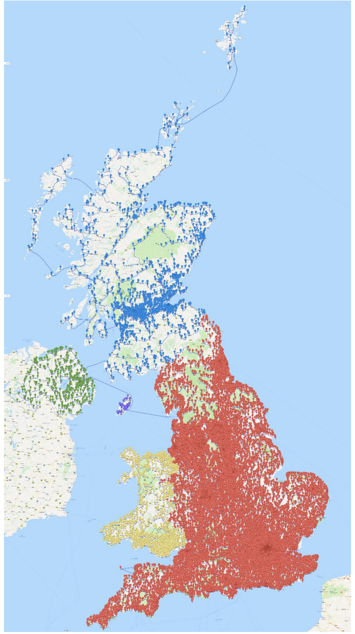
Drummer's Delight: The Shortest Way Around

FINDING the shortest route for a traveling salesman—starting from a given city, visiting each of a series of other cities, and then returning to his original point of departure—is more than an after-dinner teaser. For years it has baffled not only goods- and salesmen-routing businessmen but mathematicians as well. If a drummer visits 50 cities, for example, he has 10^{62} (62 zeros) possible itineraries. No electronic computer in existence could sort out such a large number of routes and find the shortest.

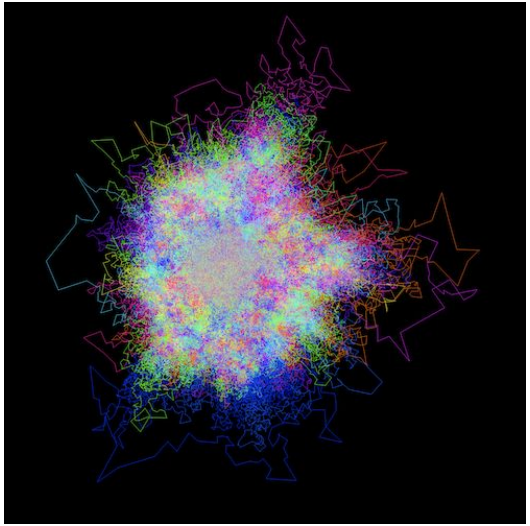
Three Rand Corp. mathematicians, using Rand McNally road-map distances between the District of Columbia and major cities in each of the 48 states, have finally produced a solution (see above). By an ingenious application of linear programming—a mathematical tool recently used to solve production-scheduling problems—it took only a few weeks for the California experts to calculate "by hand" the shortest route to cover the 49 cities: 12,345 miles.

49-city USA tour; *Newsweek*, July 26, 1954

13,509-city USA tour; Spektrum der Wissenschaft, April 1999



Optimal crawl to 49,687 pubs in the UK, March 2018



Gaia1331906450, that is 1.33B points. Current tour is 4,961,937,077 parsecs, or about 16.2 billion light years

source : https://www.math.uwaterloo.ca/tsp

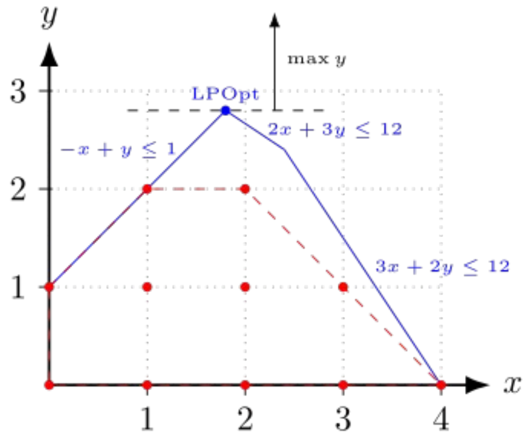
Mixed Integer Linear Programming

Objective: Optimization goal

Constraints: Business rules

Decisions: When to do how much of what

$$
\begin{aligned}
\max\ & y \\
-x + y &\le 1 \\
3x + 2y &\le 12 \\
2x + 3y &\le 12 \\
x, y &\ge 0 \\
x, y &\in \mathbb{Z}
\end{aligned}
$$



Source: https://en.wikipedia.org/wiki/Integer_programming

Optimization: Pros and cons

Pros:

- Discrete decisions (yes/no)
- Quality guarantees
- Explainable (good for reporting)
- Expressive (exploit domain)
- Deterministic (repeatability)
- Limited data need

Cons:

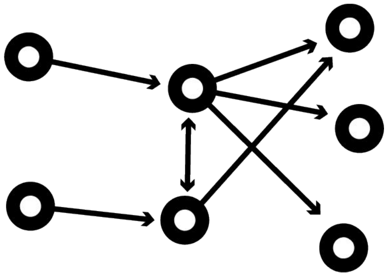
- Needs to build model
- Linear algebra skills required
- Can be time consuming
- Modelling limited by equations

Short on Network Optimization

$$
\begin{aligned}
\min \quad & \sum_{p \in P} c_p \lambda_p \\
\text{s.t.} \quad & \sum_{p \in P} a_p \lambda_p = d \\
& \lambda_p^k \geq 0, \ \ \texttt{integer}
\end{aligned}
$$

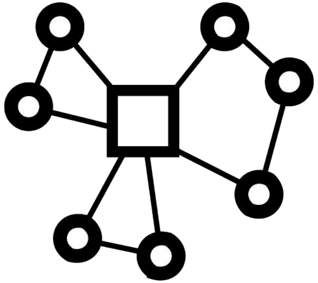
Model

Build path-based mixed integer models



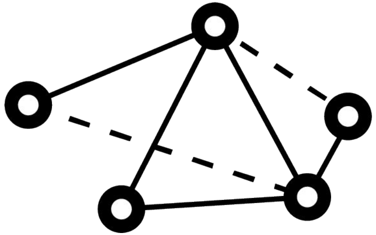
Flow

Optimize commodity flow in networks



Routing

Find optimal distribution paths



Design

Decide optimal networks

diff machine.learning math.optimization

Usage: prediction **vs** decision

Model: derives models from data **vs** need to build model using knowledge

Time: slow to train - fast to infer **vs** no training - generally slower to solve

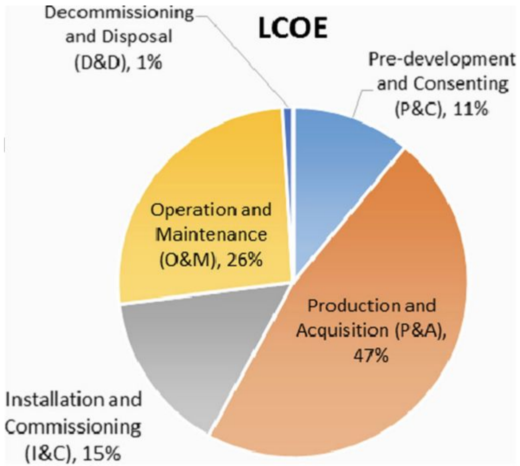
Quality: heuristic **vs** optimality guarantees

Explainability: difficult **vs** point to equation in model

Community: millions **vs** thousands

Wind Farm O&M

- Make green energy more competitive
- 26% of lifecycle cost is OPEX
- More bigger farms increases complexity
- Service provider competition leads to contractual complexity



Source: Alsubal et al. (2021), https://doi.org/10.3390/su13147943

Wind Farm O&M: Value Pool

Vessel Cost
The cost of support vessels, jack-ups, accommodation vessels

Factors:
Fleet size and mix
Contract terms

Lost Production
The power not produced due to turbine curtailment or failure

Factors:
Task scheduling
Turbine failure

ESG
Become CO2 neutral

Factors:
Fleet size and mix
Fleet utilization
Low turbine downtime

Uncertainty
Operating under high uncertainty makes planning difficult

Factors:
Weather
Turbine failure

Utilization
The utilization of technicians and support vessels

Factors:
Allocation of fleet and technicians
Task scheduling and planning
Inventory management
Disruption management

Fuel
Fuel consumption by vessels

Factors:
Routing
Speed

Wind Farm O&M: Problem Catalogue

Day-to-Day Scheduling
Detailed daily maintenance task schedule of vessel, technician, tools, personal protection equipment, spare parts.

Increase utilization.
Minimize production loss

Addresses **Utilization**, **Handling Uncertainty**, **Lost Production**, **Fuel,** and **ESG**

Medium-Term Planning
Planning of tasks on a weekly basis. Takes into account technician availability and peak periods that allows mobilizing vessels.

Capacity planning
Risk assessment

Addresses **Utilization**, **Handling Uncertainty** and **Lost Production**

Long-Term Planning
Yearly manpower and vessel capacity planning to meet scheduled and expected maintenance demand

Reduce manpower and vessel demand
Risk assessment

Addresses **Utilization**, and **Handling Uncertainty**

Spare Part Optimization
Allocation of optimal number of spare parts and tools to warehouses

Remove scheduling bottleneck
Reduce inventory

Addresses **Utilization**, **Handling Uncertainty**

Major Component Exchange
Jack-up campaign planning for changing major components

Minimize production loss
Optimize jack-up availability periods
Reduce fuel

Addresses **Handling Uncertainty**, **Lost Production**, **Vessel Cost** and **ESG**

Vessel Planning
Multi-year capacity planning and allocation. Deploy fleet efficiently and change mix depending on contractual terms and climate goals.

Reduce fleet size
Optimize acquisition costs

Addresses **Vessel Cost** and **ESG**

Short-term/operational
Days

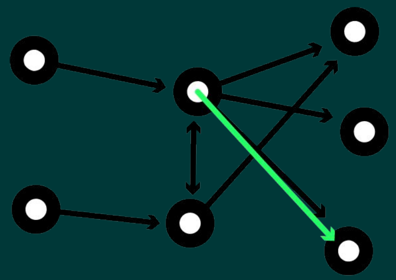
Medium-term/tactical
Weeks/months

Long-term/strategic
Year(s)

Wind Farm Major Component Exchange

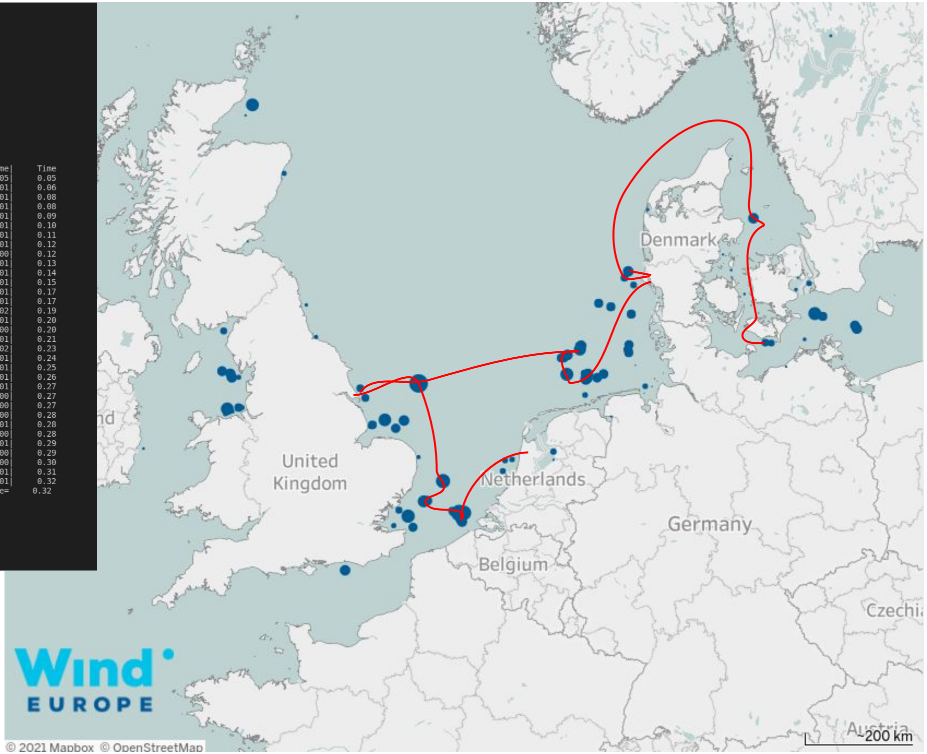
A technician scheduling like problem - plan visits at turbines for jack-ups

- **Goal:**
  - Minimize cost (acquisition, fuel, crew, lost production)
  - Time-dependent cost due to weather
- **Constraints:**
  - Vessel capacity
  - Turbine access restrictions
  - Vessel availability
- **Decision:** What vessels go where, when and do what



Wind Farm Major Component Exchange

```
Flowty Network Optimization Solver (version: 1.2.13)
License:                 Community license
User:
Expiration Date:         never

Algorithm     : PathMIP
Worker threads: 64
Parallel nodes: 32

LinearModel: columns      375| rows           8
Graphs     : num            3| edges        375

    Graph| Vertices|    Edges| Resource|     Disp|  NonDisp|       Ng|   Custom
        0|       15|      128|        4|        3|        0|        1|        0
        1|       15|      128|        4|        3|        0|        1|        0
        2|       14|      119|        4|        3|        0|        1|        0

 Node|Dpth|Relx|   It|       GUB|       GLB|       LLB|       LLP| LGap| #col|  #row| #frac| #bind| #dvar| #dcut|  LPtime| PricTime| SepaTime|  CbTime| RelxTime|    Time
    0|   0|   0|    0|       inf|      -inf|      -inf|       nan|  nan|    0|    11|     0|     8|    17|     0|    0.00|     0.05|     0.00|    0.04|     0.05|    0.05
    0|   0|   0|    1|       inf|      -inf|      -inf|       nan|  nan|   17|    11|     0|     4|     7|     0|    0.00|     0.01|     0.00|    0.01|     0.01|    0.06
    0|   0|   0|    2|       inf|      -inf|      -inf|       nan|  nan|   24|    11|     0|     7|     4|     0|    0.00|     0.01|     0.00|    0.01|     0.01|    0.08
    0|   0|   0|    3|       inf|      -inf|      -inf|       nan|  nan|   28|    11|     0|     7|     4|     0|    0.00|     0.01|     0.00|    0.00|     0.01|    0.08
    0|   0|   0|    4|       inf|      -inf|      -inf|       nan|  nan|   32|    11|     0|     7|     4|     0|    0.00|     0.01|     0.00|    0.00|     0.01|    0.09
    0|   0|   0|    5|       inf|      -inf|      -inf|       nan|  nan|   36|    11|     0|     9|     7|     0|    0.00|     0.01|     0.00|    0.00|     0.01|    0.10
    0|   0|   0|    6|       inf|      -inf|      -inf|       nan|  nan|   43|    11|     0|     5|    26|     0|    0.00|     0.01|     0.00|    0.00|     0.01|    0.11
    0|   0|   0|    7|       inf|      -inf|      -inf|       nan|  nan|   69|    11|    15|     5|     4|     0|    0.00|     0.01|     0.00|    0.01|     0.01|    0.12
    0|   0|   0|    8|       inf|      -inf|      -inf|       nan|  nan|   73|    11|    15|     6|     4|     0|    0.00|     0.00|     0.00|    0.00|     0.00|    0.12
    0|   0|   0|    9|       inf|      -inf|      -inf|       nan|  nan|   77|    11|    15|     6|     4|     0|    0.00|     0.01|     0.00|    0.00|     0.01|    0.13
    0|   0|   0|   10|       inf|      -inf|      -inf|       nan|  nan|   81|    11|    15|     7|     4|     0|    0.00|     0.01|     0.00|    0.01|     0.01|    0.14
    0|   0|   0|   11|    606702|      -inf|      -inf|    606702|  nan|   85|    11|     0|     7|    11|     0|    0.00|     0.01|     0.00|    0.01|     0.01|    0.15
    0|   0|   0|   12|    606702|      -inf|      -inf|    606702|  nan|   96|    11|     0|     8|     3|     0|    0.00|     0.01|     0.00|    0.01|     0.01|    0.17
    0|   0|   0|   13|    606702|      -inf|      -inf|    606702|  nan|   99|    11|     0|     8|    21|     0|    0.00|     0.01|     0.00|    0.00|     0.01|    0.17
    0|   0|   0|   14|    586714|      -inf|      -inf|    586714|  nan|  120|    11|     0|     8|    13|     0|    0.00|     0.02|     0.00|    0.01|     0.02|    0.19
    0|   0|   0|   15|    585251|      -inf|      -inf|    585251|  nan|  133|    11|     0|     8|    16|     0|    0.00|     0.01|     0.00|    0.00|     0.01|    0.20
    0|   0|   0|   16|    474542|      -inf|      -inf|    474542|  nan|  149|    11|     0|     9|     3|     0|    0.00|     0.00|     0.00|    0.00|     0.00|    0.20
    0|   0|   0|   17|    474542|      -inf|      -inf|    474542|  nan|  152|    11|     0|     9|     3|     0|    0.00|     0.01|     0.00|    0.00|     0.01|    0.21
    0|   0|   0|   18|    474542|      -inf|      -inf|    474542|  nan|  155|    11|     0|     9|     6|     0|    0.00|     0.02|     0.00|    0.01|     0.02|    0.23
    0|   0|   0|   19|    474542|      -inf|      -inf|    474542|  nan|  161|    11|     0|     9|     2|     0|    0.00|     0.01|     0.00|    0.00|     0.01|    0.24
    0|   0|   0|   20|    474542|      -inf|      -inf|    474542|  nan|  163|    11|     0|     9|    30|     0|    0.00|     0.01|     0.00|    0.00|     0.01|    0.25
    0|   0|   0|   21|    474542|      -inf|      -inf|    456261|  nan|  193|    11|    43|    10|     4|     0|    0.00|     0.01|     0.00|    0.00|     0.01|    0.26
    0|   0|   0|   22|    474542|      -inf|      -inf|    451085|  nan|  197|    11|    43|    10|    12|     0|    0.00|     0.01|     0.00|    0.00|     0.01|    0.27
    0|   0|   0|   23|    474542|      -inf|      -inf|    421618|  nan|  209|    11|    28|     9|     6|     0|    0.00|     0.00|     0.00|    0.00|     0.00|    0.27
    0|   0|   0|   24|    474542|      -inf|      -inf|    421618|  nan|  215|    11|    28|     9|     1|     0|    0.00|     0.00|     0.00|    0.00|     0.00|    0.27
    0|   0|   0|   25|    413579|      -inf|      -inf|    413579|  nan|  216|    11|     0|     9|     1|     0|    0.00|     0.00|     0.00|    0.00|     0.00|    0.28
    0|   0|   0|   26|    413579|      -inf|      -inf|    413579|  nan|  217|    11|     0|     9|    15|     0|    0.00|     0.00|     0.00|    0.00|     0.01|    0.28
    0|   0|   0|   27|    340038|      -inf|      -inf|    340038|  nan|  232|    11|     0|     8|     1|     0|    0.00|     0.00|     0.00|    0.00|     0.00|    0.28
    0|   0|   0|   28|    340038|      -inf|      -inf|    340038|  nan|  233|    11|     0|     9|     9|     0|    0.00|     0.01|     0.00|    0.00|     0.01|    0.29
    0|   0|   0|   29|    340038|      -inf|      -inf|    340038|  nan|  242|    11|     0|     8|     2|     0|    0.00|     0.00|     0.00|    0.00|     0.00|    0.29
    0|   0|   0|   30|    339280|      -inf|      -inf|    339280|  nan|  244|    11|     0|     9|     3|     0|    0.00|     0.00|     0.00|    0.00|     0.00|    0.30
    0|   0|   0|   31|    332770|      -inf|      -inf|    332770|  nan|  247|    11|     0|     8|     1|     0|    0.00|     0.01|     0.00|    0.00|     0.01|    0.31
    0|   0|   0|   32|    332000|      -inf|    332000|    332000|0.000|  248|    11|     0|     8|     0|     0|    0.00|     0.01|     0.00|    0.00|     0.01|    0.32
Node     0(0), Tree=    1/0, GUB=  332000, GLB=  332000, GGap=  0.000, LLB=  332000, LGap=  0.000, #col=  248, #row=   11, #frac=    0, NodeTime=   0.32, Time=    0.32

Status :        Optimal
UB     :         332000
LB     :         332000
Gap    :          0.000
NumSols:              9
Tree   :            1/0
Time   :           0.32
```



Source: https://windeurope.org/intelligence-platform/product/european-offshore-wind-farms-map-public/

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